Calculus
Graphical, Numerical, Algebraic

Ross L. Finney
Franklin D. Demana
Bert K. Waits
Daniel Kennedy

The Ohio State University
The Ohio State University
Baylor School

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Foreword

This text, as the edition before it, was especially designed and written for teachers and students of Advanced Placement Calculus. Combining the scholarship of Ross Finney and Frank Demana, the technological expertise of Bert Waits, and the intimate knowledge of and experience with the Advanced Placement Program of Dan Kennedy, this text is truly unique among calculus texts. It may be used, in perfect order and without supplementation, from the first day of the course until the day of the AP* exam. Teachers who are new to teaching calculus, as well as those who are very experienced, will be amazed at the insightful and unique treatment of many topics.

The text is a perfect balance of exploration and theory. Students are asked to explore many topics before theoretical proof. The topic of slope fields, studied at the beginning of Chapter 6 when differential equations are first introduced, has been considerably expanded. Local linearity, stressed throughout the text, permits the early introduction of l’Hôpital’s Rule. When the definite integral is introduced, students are first asked to find total change given over a specific period of time given a rate of change before they consider geometric applications. The section on logistic growth—so important in real-life situations—has been expanded. Functions are defined graphically, with tables, and with words as well as algebraically throughout the text. Problems and exercises throughout are based on real-life situations, and many are similar to questions appearing on the AP* exams. The series chapter uses technology to enhance understanding. This is a brilliant approach, and is the way that series should be presented. Students studying series from this chapter will gain a unique and thorough understanding of the topic. This textbook is one of a very few that teaches what conditional convergence means. Chapter 10, Parametric, Vector, and Polar Functions, covers vectors of two dimensions, and is perfect for students of Calculus BC. This chapter teaches exactly what the AP* student is expected to know about vector functions.

Ross Finney has passed away since this new edition was started, but his influence and scholarship are still keenly felt in the text. Throughout his life, Ross was always a master teacher, but even he was amazed at the insight and brilliance of the team of Dan, Frank, and Bert. This new edition is well prepared to take student and teacher on their journey through AP* Calculus, and I recommend it with the highest enthusiasm. There is no more comfortable, complete conveyance available anywhere.

—Judith Broadwin

Judy Broadwin taught AP* Calculus at Jericho High School for many years. In addition, she was a reader, table leader, and eventually BC Exam leader of the AP* exam. She was a member to the Development Committee for AP* Calculus during the years that the AP* course descriptions were undergoing significant change. Judy now teaches calculus at Baruch College of the City of New York.

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*Every section throughout the book also includes “Exploration” and “Extending the Ideas” features which follow the exercises.*
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About the Authors

Ross L. Finney
Ross Finney received his undergraduate degree and Ph.D. from the University of Michigan at Ann Arbor. He taught at the University of Illinois at Urbana-Champaign from 1966 to 1980 and at the Massachusetts Institute of Technology (MIT) from 1980 to 1990. Dr. Finney worked as a consultant for the Educational Development Center in Newton, Massachusetts. He directed the Undergraduate Mathematics and its Applications Project (UMAP) from 1977 to 1984 and was founding editor of the *UMAP Journal*. In 1984, he traveled with a Mathematical Association of America (MAA) delegation to China on a teacher education project through People to People International.

Dr. Finney coauthored a number of Addison-Wesley textbooks, including *Calculus; Calculus and Analytic Geometry; Elementary Differential Equations with Linear Algebra; and Calculus for Engineers and Scientists*. Dr. Finney’s coauthors were deeply saddened by the death of their colleague and friend Ross Finney on August 4, 2000.

Franklin D. Demana
Frank Demana received his master’s degree in mathematics and his Ph.D. from Michigan State University. Currently, he is Professor Emeritus of Mathematics at The Ohio State University. As an active supporter of the use of technology to teach and learn mathematics, he is cofounder of the national Teachers Teaching with Technology (T³) professional development program. He has been the director and codirector of more than $10 million of National Science Foundation (NSF) and foundation grant activities. He is currently a co-principal investigator on a $3 million grant from the U.S. Department of Education Mathematics and Science Educational Research program awarded to The Ohio State University. Along with frequent presentations at professional meetings, he has published a variety of articles in the areas of computer- and calculator-enhanced mathematics instruction. Dr. Demana is also cofounder (with Bert Waits) of the annual International Conference on Technology in Collegiate Mathematics (ICTCM). He is co-recipient of the 1997 Glenn Gilbert National Leadership Award presented by the National Council of Supervisors of Mathematics, and co-recipient of the 1998 Christofferson-Fawcett Mathematics Education Award presented by the Ohio Council of Teachers of Mathematics.


Bert K. Waits
Bert Waits received his Ph.D. from The Ohio State University and is currently Professor Emeritus of Mathematics there. Dr. Waits is cofounder of the national Teachers Teaching with Technology (T³) professional development program, and has been codirector or principal investigator on several large National Science Foundation projects. Dr. Waits has published articles in more than 50 nationally recognized professional journals. He frequently gives invited lectures, workshops, and minicourses at national meetings of the MAA and the National Council of Teachers of Mathematics (NCTM) on how to use computer technology to enhance the teaching and learning of mathematics. He has given invited presentations at the International Congress on Mathematical Education (ICME-6, -7, and -8) in Budapest (1988), Quebec (1992), and Seville (1996). Dr. Waits is co-recipient of the 1997 Glenn Gilbert National Leadership Award presented by the National Council of Supervisors of Mathematics, and is the cofounder (with Frank Demana) of the ICTCM. He is also co-recipient of the 1998 Christofferson-Fawcett Mathematics Education Award presented by the Ohio Council of Teachers of Mathematics.


Daniel Kennedy
Dan Kennedy received his undergraduate degree from the College of the Holy Cross and his master’s degree and Ph.D. in mathematics from the University of North Carolina at Chapel Hill. Since 1973 he has taught mathematics at the Baylor School in Chattanooga, Tennessee, where he holds the Cartter Lupton Distinguished Professorship. Dr. Kennedy became an Advanced Placement Calculus reader in 1978, which led to an increasing level of involvement with the program as workshop consultant, table leader, and exam leader. He joined the Advanced Placement Calculus Test Development Committee in 1986, then in 1990 became the first high school teacher in 35 years to chair that committee. It was during his tenure as chair that the program moved to require graphing calculators and laid the early groundwork for the 1998 reform of the Advanced Placement Calculus curriculum. The author of the 1997 *Teacher’s Guide—AP® Calculus*, Dr. Kennedy has conducted more than 50 workshops and institutes for high school calculus teachers. His articles on mathematics teaching have appeared in the *Mathematics Teacher* and the *American Mathematical Monthly*, and he is a frequent speaker on education reform at professional and civic meetings. Dr. Kennedy was named a Tandy Technology Scholar in 1992 and a Presidential Award winner in 1995.

Dr. Kennedy coauthored *Precalculus: Graphical, Numerical, Algebraic; Prentice Hall Algebra I; Prentice Hall Geometry; and Prentice Hall Algebra 2.*
To the Teacher

The main goal of this third edition is to realign the content with the changes in the Advanced Placement (AP*) calculus syllabus and the new type of AP* exam questions. We have also more carefully connected examples and exercises and updated the data used in examples and exercises. Cumulative Quick Quizzes are now provided two or three times in each chapter.

The course outlines for AP* Calculus reflect changes in the goals and philosophy of calculus courses now being taught in colleges and universities. The following objectives reflect the goals of the curriculum.

• Students should understand the meaning of the derivative in terms of rate of change and local linear approximations.

• Students should be able to work with functions represented graphically, numerically, analytically, or verbally, and should understand the connections among these representations.

• Students should understand the meaning of the definite integral both as a limit of Riemann sums and as a net accumulation of a rate of change, and understand the relationship between the derivative and integral.

• Students should be able to model problem situations with functions, differential equations, or integrals, and communicate both orally and in written form.

• Students should be able to represent differential equations with slope fields, solve separable differential equations analytically, and solve differential equations using numerical techniques such as Euler’s method.

• Students should be able to interpret convergence and divergence of series using technology, and to use technology to help solve problems. They should be able to represent functions with series and find the Lagrange error bound for Taylor polynomials.

This revision of Finney/Thomas/Demana/Waits Calculus completely supports the content, goals, and philosophy of the new advanced placement calculus course description.

Calculus is explored through the interpretation of graphs and tables as well as analytic methods (multiple representation of functions). Derivatives are interpreted as rates of change and local linear approximation. Local linearity is used throughout the book. The definite integral is interpreted as total change over a specific interval and as a limit of Riemann sums. Problem situations are modeled with integrals. Chapter 6 focuses on the use of differential equations to model problems. We interpret differential equations using slope fields and then solve them analytically or numerically. Convergence and divergence of series are interpreted graphically and the Lagrange error bound is used to measure the accuracy of approximating functions with Taylor polynomials.

The use of technology is integrated throughout the book to provide a balanced approach to the teaching and learning of calculus that involves algebraic, numerical, graphical, and verbal methods (the rule of four). Students are expected to use a multirepresentational approach to investigate and solve problems, to write about their conclusions, and often to work in groups to communicate mathematics orally. This book reflects what we have learned about the appropriate use of technology in the classroom during the last decade.

The visualizations and technological explorations pioneered by Demana and Waits are incorporated throughout the book. A steady focus on the goals of the advanced placement calculus curriculum has been skillfully woven into the material by Kennedy, a master high school calculus teacher. Suggestions from numerous teachers have helped us shape this modern, balanced, technological approach to the teaching and learning of calculus.

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CHANGES FOR THIS EDITION

The course descriptions for the two Advanced Placement courses (Calculus AB and Calculus BC) have changed over the years to respond to new technology and to new points of emphasis in college and university courses. The updated editions of this textbook have consistently responded to those changes to make it easier for students and teachers to adjust. This latest edition contains significantly enhanced coverage of the following topics:

- Slope fields, now a topic for both AB and BC students, are studied in greater depth and are used to visualize differential equations from the beginning.
- Euler’s method, currently a BC topic, is used as a numerical technique (with multiple examples) for solving differential equations using the insights gained from slope fields.
- Local linearity, a point of emphasis in previous editions but now more important than ever for understanding various applications of the derivative, is now a thread running throughout the book.
- More examples and exercises have been added to illustrate the connections between the graph of a function and the graph of its derivative (or the graph of f and a function defined as an integral of f).
- The logistic differential equation, a BC topic that is covered weakly in most textbooks despite many applications, now has its own section.

Similarly, the coverage of some other topics has been trimmed to reflect the intent of their inclusion in the AP* courses:

- The use of partial fractions for finding antiderivatives has been narrowed to distinct linear factors in the denominator and has been more directly linked to the logistic differential equation;
- The treatment of vector calculus has been revised to focus on planar motion problems, which are easily solved using earlier results componentwise;
- The treatment of polar functions has been narrowed to the polar topics in the BC course description and has been linked more directly to the treatment of parametric functions.

Moreover, this latest edition continues to explore the ways teachers and students can use graphing calculator technology to enhance their understanding of calculus topics.

This edition of the text also includes new features to further assist students in their study of calculus:

- What You’ll Learn About… and Why introduces the big ideas in each section and explains their purpose.
- At the end of each example students are encouraged to Now Try a related exercise at the end of the section to check their comprehension.
- A Quick Quiz for AP* Preparation appears every few sections, requiring students to answer questions about topics covered in multiple sections, to assist them in obtaining a conceptual understanding of the material.
- Each exercise set includes a group of Standardized Test Questions. Additionally, an AP* Examination Preparation appears at the end of each set of chapter review exercises.

For further information about new and continuing features, please consult the To the Student material.
CONTINUING FEATURES

Balanced Approach
A principal feature of this edition is the balance attained among the rule of four: analytic/algebraic, numerical, graphical, and verbal methods of representing problems. We believe that students must value all of these methods of representation, understand how they are connected in a given problem, and learn how to choose the one(s) most appropriate for solving a particular problem.

The Rule of Four
In support of the rule of four, we use a variety of techniques to solve problems. For instance, we obtain solutions algebraically or analytically, support our results graphically or numerically with technology, and then interpret the result in the original problem context. We have written exercises where students are asked to solve problems by one method and then support or confirm their solutions by using another method. We want students to understand that technology can be used to support (but not prove) results, and that algebraic or analytic techniques are needed to prove results. We want students to understand that mathematics provides the foundation that allows us to use technology to solve problems.

Applications
The text includes a rich array of interesting applications from biology, business, chemistry, economics, engineering, finance, physics, the social sciences, and statistics. Some applications are based on real data from cited sources. Students are exposed to functions as mechanisms for modeling data and learn about how various functions can model real-life problems. They learn to analyze and model data, represent data graphically, interpret from graphs, and fit curves. Additionally, the tabular representations of data presented in the text highlight the concept that a function is a correspondence between numerical variables, helping students to build the connection between the numbers and the graphs.

Explorations
Students are expected to be actively involved in understanding calculus concepts and solving problems. Often the explorations provide a guided investigation of a concept. The explorations help build problem-solving ability by guiding students to develop a mathematical model of a problem, solve the mathematical model, support or confirm the solution, and interpret the solution. The ability to communicate their understanding is just as important to the learning process as reading or studying, not only in mathematics but in every academic pursuit. Students can gain an entirely new perspective on their knowledge when they explain what they know in writing.

Graphing Utilities
The book assumes familiarity with a graphing utility that will produce the graph of a function within an arbitrary viewing window, find the zeros of a function, compute the derivative of a function numerically, and compute definite integrals numerically. Students are expected to recognize that a given graph is reasonable, identify all the important characteristics of a graph, interpret those characteristics, and confirm them using analytic methods. Toward that end, most graphs appearing in this book resemble students' actual grapher output or suggest hand-drawn sketches. This is one of the first calculus textbooks to take full advantage of graphing calculators, philosophically restructuring the course to teach new things in new ways to achieve new understanding, while (courageously) abandoning some old things and old ways that are no longer serving a purpose.
Exercise Sets
The exercise sets were revised extensively for this edition, including many new ones. There are nearly 4,000 exercises, with more than 80 Quick Quiz exercises and 560 Quick Review exercises. The different types of exercises included are:
- Algebraic and analytic manipulation
- Interpretation of graphs
- Graphical representations
- Numerical representations
- Explorations
- Writing to learn
- Group activities
- Data analyses
- Descriptively titled applications
- Extending the ideas

Each exercise set begins with the Quick Review feature, which can be used to introduce lessons, support Examples, and review prerequisite skills. The exercises that follow are graded from routine to challenging. An additional block of exercises, Extending the Ideas, may be used in a variety of ways, including group work. We also provide Review Exercises and AP* Examination Preparation at the end of each chapter.

SUPPLEMENTS AND RESOURCES
For the Student
- Introduction to the AP* AB and BC Calculus Exams
- Precalculus Review of Calculus Prerequisites
- Review of AP* Calculus AB and Calculus BC Topics
- Practice Exams
- Answers and Solutions

Student Practice Workbook, ISBN 0-13-201411-4
- New examples that parallel key examples from each section in the book are provided along with a detailed solution
- Related practice problems follow each example

- An introduction to Texas Instruments’ graphing calculators, as they are used for calculus

For the Teacher
- Answers included on the same page as the problem appears, for most exercises
• Solutions to Chapter Opening Problems, Teaching Notes, Common Errors, Notes on Examples and Exploration Extensions, and Assignment Guide included at the beginning of the book.

**Teacher’s AP* Correlations and Preparation Guide, 0-13-201413-0**

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• Complete solutions for Quick Reviews, Exercises, Explorations, and Chapter Reviews


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graphs, import graphics, and insert math notation, variable numbers, or text. Tests can be printed or administered online via the Internet or another network. TestGen comes packaged with QuizMaster, which allows students to take tests on a local area network. The software is available on a dual-platform Windows/Macintosh CD-ROM.

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Available for download from the PHSchool.com Web site (http://www.phschool.com/). Enter the code aze-0002 in the Web Codes box in the upper-left corner of the home page. Please note the Web Code is case sensitive.

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**To the AP* Student**

We know that as you study for your AP* course, you’re preparing along the way for the AP* exam. By tying the material in this book directly to AP* course goals and exam topics, we help you to focus your time most efficiently. And that’s a good thing!

The AP* exam is an important milestone in your education. A high score will position you optimally for college acceptance—and possibly will give you college credits that put you a step ahead. Our primary commitment is to provide you with the tools you need to excel on the exam ... the rest is up to you!

**Test-Taking Strategies for an Advanced Placement* Calculus Examination**

You should approach the AP* Calculus Examination the same way you would any major test in your academic career. Just remember that it is a one-shot deal—you should be at your peak performance level on the day of the test. For that reason you should do everything that your “coach” tells you to do. In most cases your coach is your classroom teacher. It is very likely that your teacher has some experience, based on workshop information or previous students’ performance, to share with you.

You should also analyze your own test-taking abilities. At this stage in your education, you probably know your strengths and weaknesses in test-taking situations. You may be very good at multiple choice questions but weaker in essays, or perhaps it is the other way around. Whatever your particular abilities are, evaluate them and respond accordingly. Spend more time on your weaker points. In other words, rather than spending time in your comfort zone where you need less work, try to improve your soft spots. In all cases, concentrate on clear communication of your strategies, techniques, and conclusions.
The following table presents some ideas in a quick and easy form.

### General Strategies for AP* Examination Preparation

<table>
<thead>
<tr>
<th>Time</th>
<th>Dos</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Through the Year</strong></td>
<td>• Register with your teacher/coordinator</td>
</tr>
<tr>
<td></td>
<td>• Pay your fee (if applicable) on time</td>
</tr>
<tr>
<td></td>
<td>• Take good notes</td>
</tr>
<tr>
<td></td>
<td>• Work with others in study groups</td>
</tr>
<tr>
<td></td>
<td>• Review on a regular basis</td>
</tr>
<tr>
<td></td>
<td>• Evaluate your test-taking strengths and weaknesses—keep track of how successful you are when guessing</td>
</tr>
<tr>
<td><strong>The Week Before</strong></td>
<td>• Combine independent and group review</td>
</tr>
<tr>
<td></td>
<td>• Get tips from your teacher</td>
</tr>
<tr>
<td></td>
<td>• Do lots of mixed review problems</td>
</tr>
<tr>
<td></td>
<td>• Check your exam date, time, and location</td>
</tr>
<tr>
<td></td>
<td>• Review the appropriate AP* Calculus syllabus (AB or BC)</td>
</tr>
<tr>
<td><strong>The Night Before</strong></td>
<td>• Put new batteries in your calculator</td>
</tr>
<tr>
<td></td>
<td>• Make sure your calculator is on the approved list</td>
</tr>
<tr>
<td></td>
<td>• Lay out your clothes and supplies so that you are ready to go out the door</td>
</tr>
<tr>
<td></td>
<td>• Do a short review</td>
</tr>
<tr>
<td></td>
<td>• Go to bed at a reasonable hour</td>
</tr>
<tr>
<td><strong>Exam Day</strong></td>
<td>• Get up a little earlier than usual</td>
</tr>
<tr>
<td></td>
<td>• Eat a good breakfast/lunch</td>
</tr>
<tr>
<td></td>
<td>• Put some hard candy in your pocket in case you need an energy boost during the test</td>
</tr>
<tr>
<td></td>
<td>• Get to your exam location 15 minutes early</td>
</tr>
<tr>
<td><strong>Exam Night</strong></td>
<td>• Relax—you earned it</td>
</tr>
</tbody>
</table>

### Topics from the Advanced Placement* Curriculum for Calculus AB, Calculus BC

As an AP* Student, you are probably well aware of the good study habits that are needed to be a successful student in high school and college:

- attend all the classes
- ask questions (either during class or after)
- take clear and understandable notes
- make sure you understand the concepts rather than memorizing formulas
- do your homework; extend your test-prep time over several days or weeks, instead of cramming
- use all the resources—text and people—that are available to you.

No doubt this list of “good study habits” is one that you have seen or heard before. You should know that there is powerful research that suggests a few habits or routines will enable you to go beyond “knowing about” calculus, to more deeply “understanding” calculus. Here are three concrete actions for you to consider:

- Review your notes at least once a week and rewrite them in summary form.
- Verbally explain concepts (theorems, etc.) to a classmate.
• Form a study group that meets regularly to do homework and discuss reading and lecture notes.

Most of these tips boil down to one mantra, which all mathematicians believe in:

*Math is not a spectator sport.*

The AP* Calculus Examination is based on the following Topic Outline. For your convenience, we have noted all Calculus AB and Calculus BC objectives with clear indications of topics required only by the Calculus BC Exam. The outline cross-references each AP* Calculus objective with the appropriate section(s) of this textbook: *Calculus: Graphical, Numerical, Algebraic*, Third Edition, by Finney, Demana, Waits, and Kennedy.

Use this outline to track your progress through the AP* exam topics. Be sure to cover every topic associated with the exam you are taking. Check it off when you have studied and/or reviewed the topic.

Even as you prepare for your exam, I hope this book helps you map—and enjoy—your calculus journey!

—John Brunsting

Hinsdale Central High School

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**Topic Outline for AP* Calculus AB and AP* Calculus BC**

(excerpted from the College Board's Course Description - Calculus: Calculus AB, Calculus BC, May 2007)

<table>
<thead>
<tr>
<th>I.</th>
<th>Calculus Exam</th>
<th>Functions, Graphs, and Limits</th>
<th>Calculus</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>AB</td>
<td>AB</td>
<td>1.2–1.6</td>
</tr>
<tr>
<td>B</td>
<td>AB</td>
<td>BC</td>
<td>Limits of functions (including one-sided limits)</td>
</tr>
<tr>
<td>B1</td>
<td>AB</td>
<td>BC</td>
<td>An intuitive understanding of the limiting process</td>
</tr>
<tr>
<td>B2</td>
<td>AB</td>
<td>BC</td>
<td>Calculating limits using algebra</td>
</tr>
<tr>
<td>B3</td>
<td>AB</td>
<td>BC</td>
<td>Estimating limits from graphs or tables of data</td>
</tr>
<tr>
<td>C</td>
<td>AB</td>
<td>BC</td>
<td>Asymptotic and unbounded behavior</td>
</tr>
<tr>
<td>C1</td>
<td>AB</td>
<td>BC</td>
<td>Understanding asymptotes in terms of graphical behavior</td>
</tr>
<tr>
<td>C2</td>
<td>AB</td>
<td>BC</td>
<td>Describing asymptotic behavior in terms of limits involving infinity</td>
</tr>
<tr>
<td>C3</td>
<td>AB</td>
<td>BC</td>
<td>Comparing relative magnitudes of functions and their rates of change</td>
</tr>
<tr>
<td>D</td>
<td>AB</td>
<td>BC</td>
<td>Continuity as a property of functions</td>
</tr>
<tr>
<td>D1</td>
<td>AB</td>
<td>BC</td>
<td>An intuitive understanding of continuity</td>
</tr>
<tr>
<td>D2</td>
<td>AB</td>
<td>BC</td>
<td>Understanding continuity in terms of limits</td>
</tr>
<tr>
<td>D3</td>
<td>AB</td>
<td>BC</td>
<td>Geometric understanding of graphs of continuous functions</td>
</tr>
<tr>
<td>E</td>
<td>BC</td>
<td>BC</td>
<td>Parametric, polar, and vector functions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>II.</th>
<th>Calculus Exam</th>
<th>Derivatives</th>
<th>Calculus</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>AB</td>
<td>AB</td>
<td>Concept of the derivative</td>
</tr>
<tr>
<td>A1</td>
<td>AB</td>
<td>BC</td>
<td>Derivative presented graphically, numerically, and analytically</td>
</tr>
<tr>
<td>A2</td>
<td>AB</td>
<td>BC</td>
<td>Derivative interpreted as an instantaneous rate of change</td>
</tr>
<tr>
<td>A3</td>
<td>AB</td>
<td>BC</td>
<td>Derivative defined as the limit of the difference quotient</td>
</tr>
<tr>
<td>A4</td>
<td>AB</td>
<td>BC</td>
<td>Relationship between differentiability and continuity</td>
</tr>
<tr>
<td>B</td>
<td>AB</td>
<td>AB</td>
<td>Derivative at a point</td>
</tr>
<tr>
<td>B1</td>
<td>AB</td>
<td>BC</td>
<td>Slope of a curve at a point</td>
</tr>
<tr>
<td>B2</td>
<td>AB</td>
<td>BC</td>
<td>Tangent line to a curve at a point and local linear approximation</td>
</tr>
<tr>
<td>B3</td>
<td>AB</td>
<td>BC</td>
<td>Instantaneous rate of change as the limit of average rate of change</td>
</tr>
<tr>
<td>B4</td>
<td>AB</td>
<td>BC</td>
<td>Approximate rate of change from graphs and tables of values</td>
</tr>
<tr>
<td>C</td>
<td>AB</td>
<td>BC</td>
<td>Derivative as a function</td>
</tr>
<tr>
<td>C1</td>
<td>AB</td>
<td>BC</td>
<td>Corresponding characteristics of graphs of $f$ and $f'$</td>
</tr>
</tbody>
</table>

---
### C2 AB BC Relationship between the increasing and decreasing behavior of $f$ and the sign of $f'$. 4.1, 4.3

### C3 AB BC The Mean Value Theorem and its geometric consequences. 4.2

### C4 AB BC Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa 3.4, 3.5, 4.6, 6.4, 6.5

### D AB BC Second Derivatives

#### D1 AB BC Corresponding characteristics of graphs of $f, f'$ and $f''$. 4.3

#### D2 AB BC Relationship between the concavity of $f$ and the sign of $f''$. 4.3

#### D3 AB BC Points of inflection as places where concavity changes. 4.3

### E AB BC Applications of derivatives

#### E1 AB BC Analysis of curves, including the notions of monotonicity and concavity. 4.1–4.3

#### E2 BC Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors. 10.1–10.3

#### E3 AB BC Optimization, both absolute (global) and relative (local) extrema. 4.3, 4.4

#### E4 AB BC Modeling rates of change, including related rates problems. 4.6

#### E5 AB BC Use of implicit differentiation to find the derivative of an inverse function. 3.7

#### E6 AB BC Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration. 3.4

#### E7 AB BC Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations. 6.1

#### E8 BC Numerical solution of differential equations using Euler's method. 6.1

#### E9 BC L'Hopital's Rule, including its use in determining limits and convergence of improper integrals and series. 8.1, 9.5

### F AB BC Computation of derivatives

#### F1 AB BC Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions. 3.3, 3.5, 3.8, 3.9

#### F2 AB BC Basic rules for the derivative of sums, products, and quotients of functions. 3.3

#### F3 AB BC Chain rule and implicit differentiation. 3.6, 3.7

#### F4 BC Derivatives of parametric, polar, and vector functions. 10.1–10.3

### III. Calculus Exam

#### Integrals

### A AB BC Interpretations and properties of definite integrals

#### A1 AB BC Definite integral as a limit of Riemann sums. 5.1, 5.2

#### A2 AB BC Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the closed interval $[a,b]$ of $f'(x)dx = f(b) - f(a)$. 5.1, 5.4

#### A3 AB BC Basic properties of definite integrals (Examples include additivity and linearity). 5.2 - 5.3

### B AB BC Applications of integrals

#### B1a AB BC Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Students should be able to adapt their knowledge and techniques. Emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. Specific applications should include using the integral of a rate of change to give accumulated change, finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line. 5.4, 5.5, 6.4, 6.5, 7.1–7.5

#### B1b BC Appropriate integrals are used ... specific applications should include ... finding the area of a region bounded by polar curves ... and the length of a curve (including a curve given in parametric form). 7.4, 10.1, 10.3

### C AB BC Fundamental Theorem of Calculus

#### C1 AB BC Use of the Fundamental Theorem to evaluate definite integrals. 5.4

#### C2 AB BC Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so derived. 5.4, 6.1
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>Techniques of antidifferentiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>AB</td>
<td>BC</td>
<td>Antiderivatives following directly from derivatives of basic functions 4.2, 6.1, 6.2</td>
</tr>
<tr>
<td>D2a</td>
<td>AB</td>
<td>BC</td>
<td>Antiderivatives by substitution of variables (including change of limits for definite integrals) 6.2</td>
</tr>
<tr>
<td>D2b</td>
<td>BC</td>
<td></td>
<td>Antiderivatives by ... parts, and simple partial fractions (nonrepeating linear factors only) 6.3, 6.5</td>
</tr>
<tr>
<td>D3</td>
<td>BC</td>
<td></td>
<td>Improper integrals (as limits of definite integrals) 8.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E</th>
<th>Applications of antidifferentiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>AB</td>
</tr>
<tr>
<td>E2</td>
<td>AB</td>
</tr>
<tr>
<td>E3</td>
<td>BC</td>
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</table>

<table>
<thead>
<tr>
<th>F</th>
<th>Numerical approximations to definite integrals</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>AB</td>
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</table>

<table>
<thead>
<tr>
<th>IV.</th>
<th>Calculus Exam</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Concept of series</td>
</tr>
<tr>
<td>A1</td>
<td>BC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B</th>
<th>Series of constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>BC</td>
</tr>
<tr>
<td>B2</td>
<td>BC</td>
</tr>
<tr>
<td>B3</td>
<td>BC</td>
</tr>
<tr>
<td>B4</td>
<td>BC</td>
</tr>
<tr>
<td>B5</td>
<td>BC</td>
</tr>
<tr>
<td>B6</td>
<td>BC</td>
</tr>
<tr>
<td>B7</td>
<td>BC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C</th>
<th>Taylor series</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>BC</td>
</tr>
<tr>
<td>C2</td>
<td>BC</td>
</tr>
<tr>
<td>C3</td>
<td>BC</td>
</tr>
<tr>
<td>C4</td>
<td>BC</td>
</tr>
<tr>
<td>C5</td>
<td>BC</td>
</tr>
<tr>
<td>C6</td>
<td>BC</td>
</tr>
<tr>
<td>C7</td>
<td>BC</td>
</tr>
</tbody>
</table>
Using the Book for Maximum Effectiveness

So, how can this book help you to join in the game of mathematics for a winning future? Let us show you some unique tools that we have included in the text to help prepare you not only for the AP* Calculus exam, but also for success beyond this course.

Chapter Openers provide a motivating photograph and application to show you an example that illustrates the relevance of what you’ll be learning in the chapter.

A Chapter Overview then follows to give you a sense of what you are going to learn. This overview provides a roadmap of the chapter as well as tells how the different topics in the chapter are connected under one big idea. It is always helpful to remember that mathematics isn’t modular, but interconnected, and that the different skills you are learning throughout the course build on one another to help you understand more complex concepts.

Similarly, the What you’ll learn about…and why feature gives you the big ideas in each section and explains their purpose. You should read this as you begin the section and always review it after you have completed the section to make sure you understand all of the key topics that you have just studied.

Margin Notes appear throughout the book on various topics. Some notes provide more information on a key concept or an example. Other notes offer practical advice on using your graphing calculator to obtain the most accurate results.

Brief Historical Notes present the stories of people and the research that they have done to advance the study of mathematics. Reading these notes will often provide you with additional insight for solving problems that you can use later when doing the homework or completing the AP* Exam.

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EXAMPLE 3 Fabricating a Box

An open box made by cut-out squares that are about 3.68 in. on a side give the maximum volume.

Interpret

Cutout squares that are about 3.68 in. on a side gives the maximum volume.

Confirm Analytically

Because 2 maximum volume?

Because 2 maximum volume?

Solve Graphically

Because 2 maximum volume?

Solve Numerically

Because 2 maximum volume?

Many examples include solutions to Solve Algebraically, Solve Graphically, or Solve Numerically. You should be able to use different approaches for finding solutions to problems. For instance, you would obtain a solution algebraically when that is the most appropriate technique to use, and you would obtain solutions graphically or numerically when algebra is difficult or impossible to use. We urge you to solve problems by one method, then support or confirm your solution by using another method, and finally, interpret the results in the context of the problem. Doing so reinforces the idea that to understand a problem fully, you need to understand it algebraically, graphically, and numerically whenever possible.

Each example ends with a suggestion to Now Try a related exercise. Working the suggested exercise is an easy way for you to check your comprehension of the material while reading each section, instead of waiting until the end of each section or chapter to see if you "got it." True comprehension of the textbook is essential for your success on the AP* Exam.

Explorations appear throughout the text and provide you with the perfect opportunity to become an active learner and discover mathematics on your own. Honing your critical thinking and problem-solving skills will ultimately benefit you on all of your AP* Exams.

Each exercise set begins with a Quick Review to help you review skills needed in the exercise set, reminding you again that mathematics is not modular. Each Quick Review includes section references to show where these skills were covered earlier in the text. If you find these problems overly challenging, you should go back through the book and your notes to review the material covered in previous chapters. Remember, you need to understand the material from the entire calculus course for the AP* Calculus Exam, not just memorize the concepts from the last part of the course.

Quick Review 6.3 (For help, go to Sections 3.8 and 3.9.)

1. y = \sin 3x
2. y = \sin \ln (3x + 1)
3. y = \tan^{-1} 2x
4. y = \sin^{-1} (x + 3)

5. y = \tan^{-1} 3x
6. y = \cos^{-1} (x + 1)
7. Find the area under the arch of the curve y = \sin mx from x = 0 to x = 1.

8. Solve the differential equation dy/dx = e^x.
9. Solve the initial value problem dy/dx = x + \sin x, y(0) = 2.
10. Use differentiation to confirm the integration formula

\[ \int x^2 \sin x \, dx = \frac{1}{2} x^2 \sin x - \frac{1}{2} x \cos x. \]

Along with the standard types of exercises, including skill-based, application, writing, exploration, and extension questions, each exercise set includes a group of Standardized Test Questions. Each group includes two true-false with justifications and four multiple-choice questions, with instructions about the permitted use of your graphing calculator.
An AP® Examination Preparation section appears at the end of each set of chapter review exercises and includes three free-response questions of the AP® type. This set of questions, which also may or may not permit the use of your graphing calculator, gives you additional opportunity to practice skills and problem-solving techniques needed for the AP® Calculus Exam.

Calculus at Work features individuals who are using calculus in their jobs, providing you with some insight as to when you will use calculus in your careers. Some of the applications of calculus they encounter are mentioned throughout the text.

Each chapter concludes with a list of Key Terms, with references back to where they are covered in the chapter, as well as Chapter Review Exercises to check your comprehension of the chapter material.

The Quick Quiz for AP® Preparation provides another opportunity to review your understanding as you progress through each chapter. A quiz appears after every two or three sections and asks you to answer questions about topics covered in those sections. Each quiz contains three multiple-choice questions and one free-response question of the AP® type. This continual reinforcement of ideas steers you away from rote memorization and toward the conceptual understanding needed for the AP® Calculus Exam.

To the AP® Student

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Note to Instructors:
The Annotated Teacher’s Edition contains answers to all exercises. Where space permits, answers appear on the same page as their corresponding exercises—either adjacent to the exercise or at the top or bottom of the exercise set—or answers appear in available space on the adjacent page. Where space does not permit answers to appear in the exercise set, those answers can be found in the back of the book.